$Q\bar{Q}$ pair production in high-energy hadronic interactions

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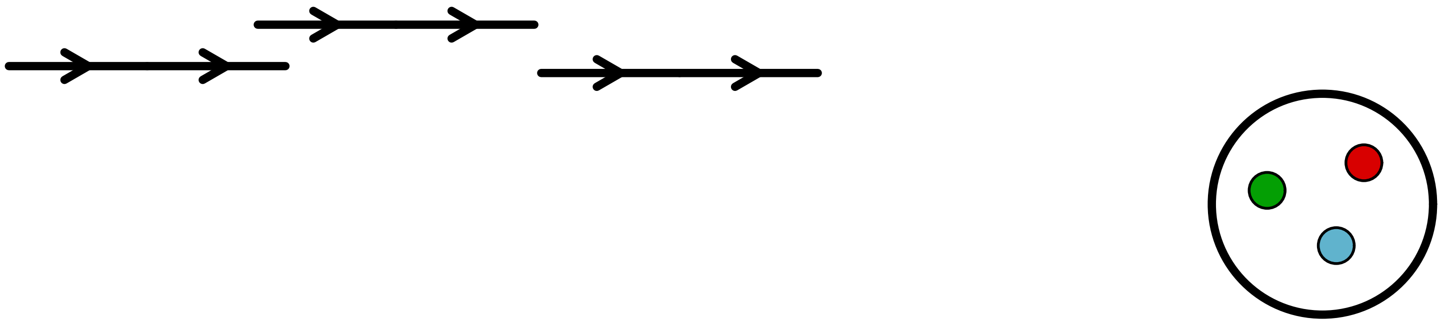
Outline

- Parton evolution and saturation
- Color Glass Condensate
- Overview of pair production
- pp collisions
- pA collisions
- AA collisions

Based on:

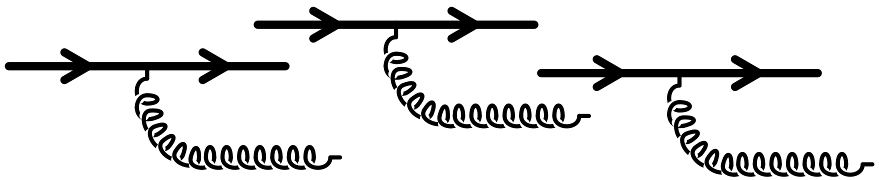
- FG, Venugopalan, hep-ph/0310090
- Blaizot, FG, Venugopalan, work in progress
- FG, Kajantie, Lappi, work in progress

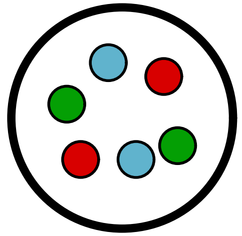
Evolution and saturation (1/5)



▷ at low energy, only valence quarks are present in the hadron wave function

Evolution and saturation (2/5)



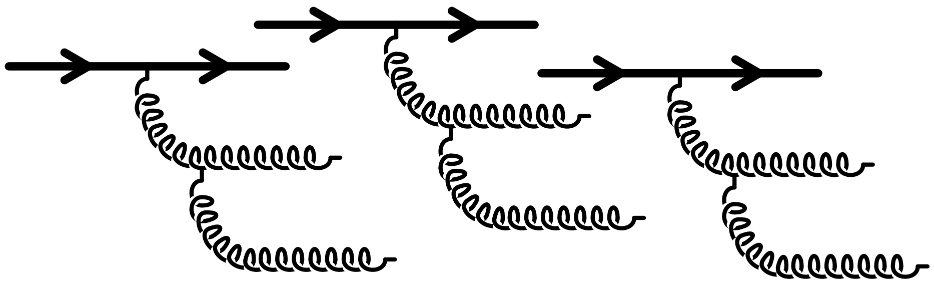


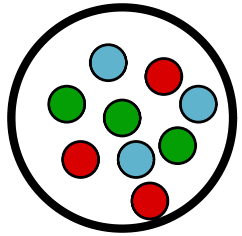
▷ when energy increases, new partons are emitted

▷ the emission probability is $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(\frac{1}{x})$, with x the longitudinal momentum fraction of the gluon

▷ at small-x (i.e. high energy), these logs need to be resummed

Evolution and saturation (3/5)

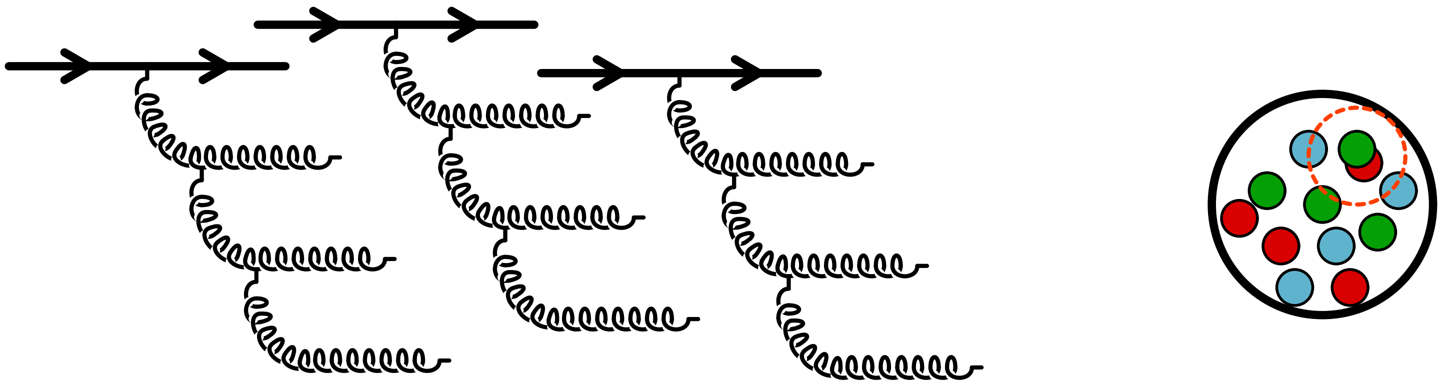




▷ as long as the density of constituents remains small, the evolution is linear: the number of partons produced at a given step is proportional to the number of partons at the previous step (BFKL)

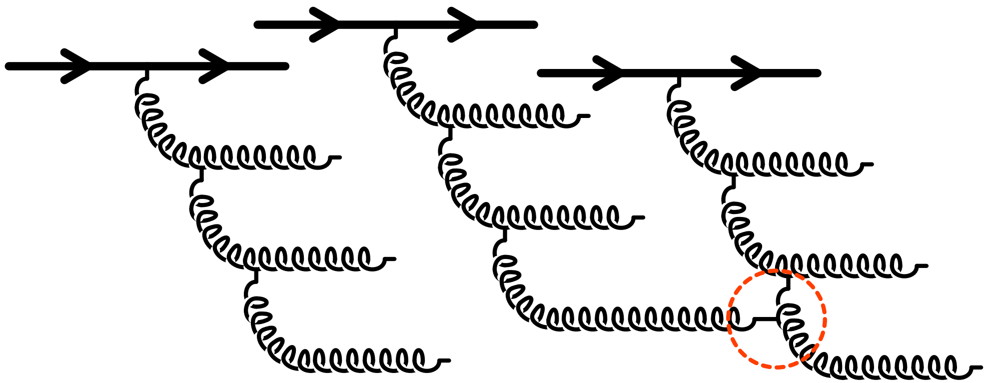
Kuraev, Lipatov, Fadin (1977), Balitsky, Lipatov (1978)

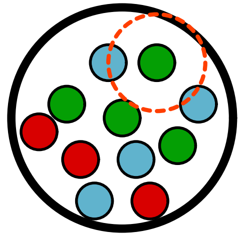
Evolution and saturation (4/5)



▷ eventually, the partons start overlapping in phase-space

Evolution and saturation (5/5)





▷ parton recombination becomes favorable

▷ after this point, the evolution is non-linear:

the number of partons created at a given step depends non-linearly on the number of partons present previously

Balitsky (1996), Kovchegov (1996,2000)

Jalilian-Marian, Kovner, Leonidov, Weigert (1997,1999)

Iancu, Leonidov, McLerran (2001)

The color glass condensate (1/2)

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- The color sources ρ_a are random, and described by a distribution functional $W_{x_0}[\rho]$, with x_0 the separation between "small-x" and "large-x".

The color glass condensate (2/2)

- Observables are calculated in the classical field, and then averaged over the hard sources ρ_a:

$$\boxed{\mathcal{O} = \int [D\rho_a]\; W_{x_0}[\rho_a]\; \mathcal{O}[\rho_a]}$$

Note: this average restores gauge invariance

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- The distribution $W_{x_0}[\rho_a]$ evolves with x_0 (more modes are included in W as x_0 decreases):

$$\frac{\partial W_{x_0}[\rho]}{\partial \ln(1/x_0)} = \frac{1}{2}\int_{\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp} \frac{\delta}{\delta\rho_a(\vec{\boldsymbol{x}}_\perp)} \chi_{ab}(\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp) \frac{\delta}{\delta\rho_b(\vec{\boldsymbol{y}}_\perp)} W_{x_0}[\rho]$$

$\triangleright$ χ_{ab} depends on ρ to all orders $\Rightarrow$ non linear evolution
$\triangleright$ reduces to BFKL in the limit of low densities

Issues in $Q\bar{Q}$ production

- Does $k_\perp$-factorization hold ? Can we hide rescatterings effects in the “unintegrated gluon distribution” ? Is this function universal ?

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- In the Color Glass Condensate picture, there are no quarks initially
  - How do we go from this quark-poor system to a system with chemically equilibrated quarks?
  - How long does it take to produce the quarks?
  - How many quarks are produced?
  - Is quark production affected by saturation?

$Q\bar{Q}$ production in a field $A(t)$

Baltz, FG, McLerran, Peshier (2001)

- Probability to create (exactly) one pair:

$$P_1 = |\langle 0_{\rm in}|0_{\rm out}\rangle|^2 \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^3 2E_{\boldsymbol{p}}} \frac{d^3\vec{\boldsymbol{q}}}{(2\pi)^3 2E_{\boldsymbol{q}}} \left|\overline{u}(\vec{\boldsymbol{q}})\mathcal{T}_{_F} v(\vec{\boldsymbol{p}})\right|^2$$

▷ $\mathcal{T}_{_F}$ is the t-ordered quark propagator in the external field

▷ $\langle 0_{\rm in}|0_{\rm out}\rangle$ is the vacuum-to-vacuum transition amplitude (sum of vacuum diagrams), required by unitarity

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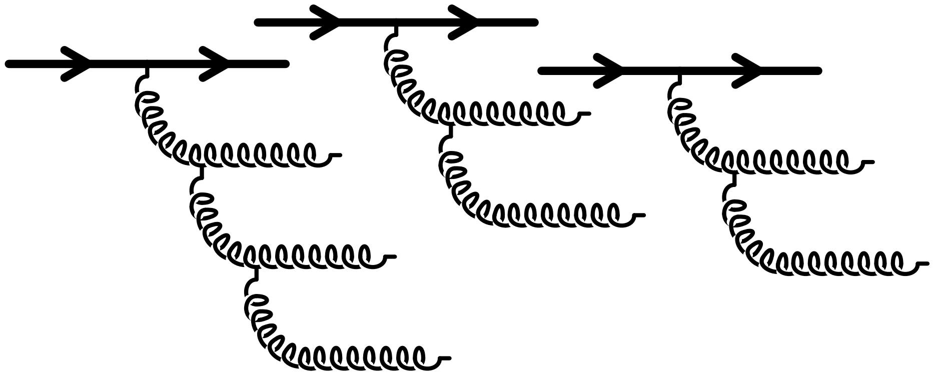
- Average pair multiplicity:

$$\overline{n} = \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^3 2E_{\boldsymbol{p}}} \frac{d^3\vec{\boldsymbol{q}}}{(2\pi)^3 2E_{\boldsymbol{q}}} \left|\overline{u}(\vec{\boldsymbol{q}})\mathcal{T}_{_R} v(\vec{\boldsymbol{p}})\right|^2$$

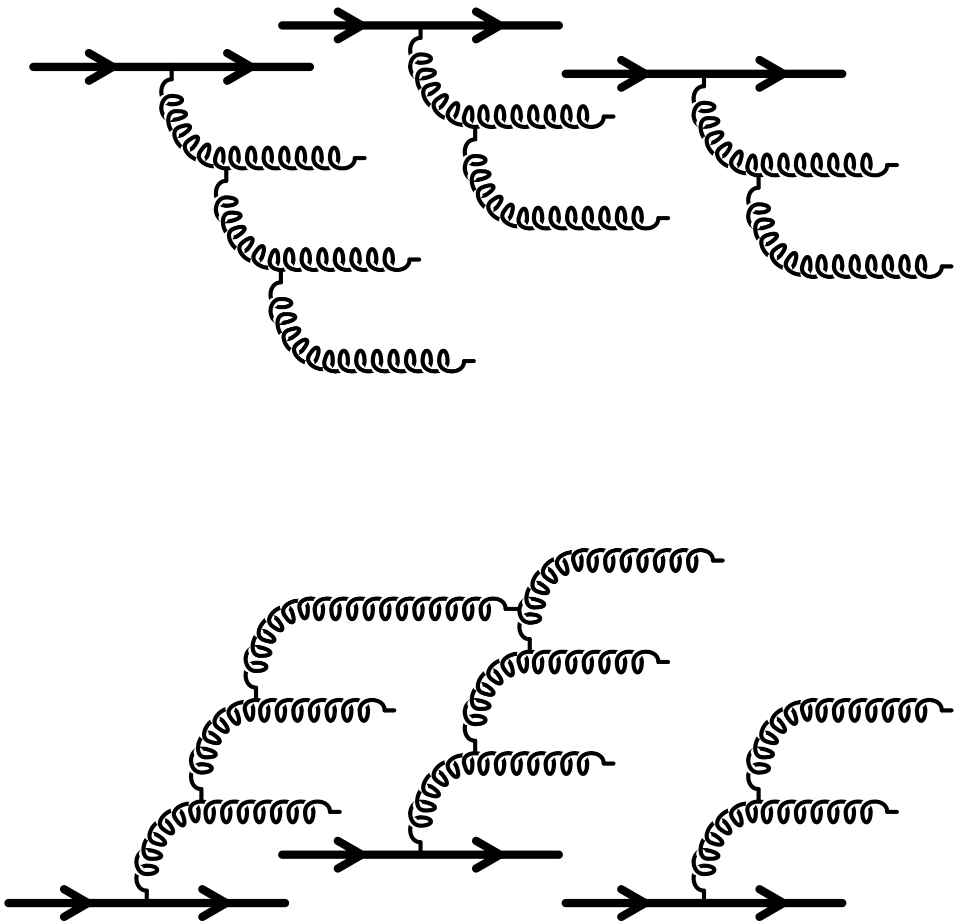
▷ $\mathcal{T}_{_R}$ is the retarded quark propagator in the external field

$Q\bar{Q}$ production: overview (1/4)



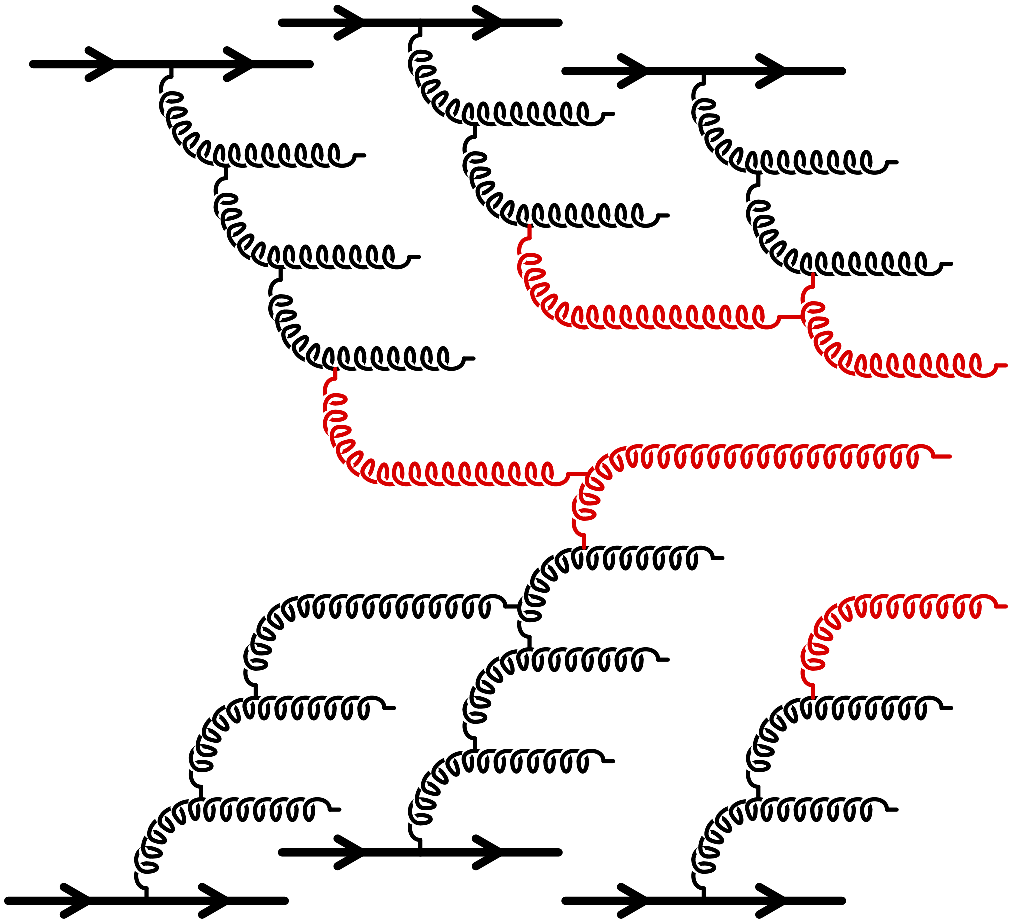
▷ find $W_{x_1}[\rho_1]$ for the first projectile

$Q\bar{Q}$ production: overview (2/4)



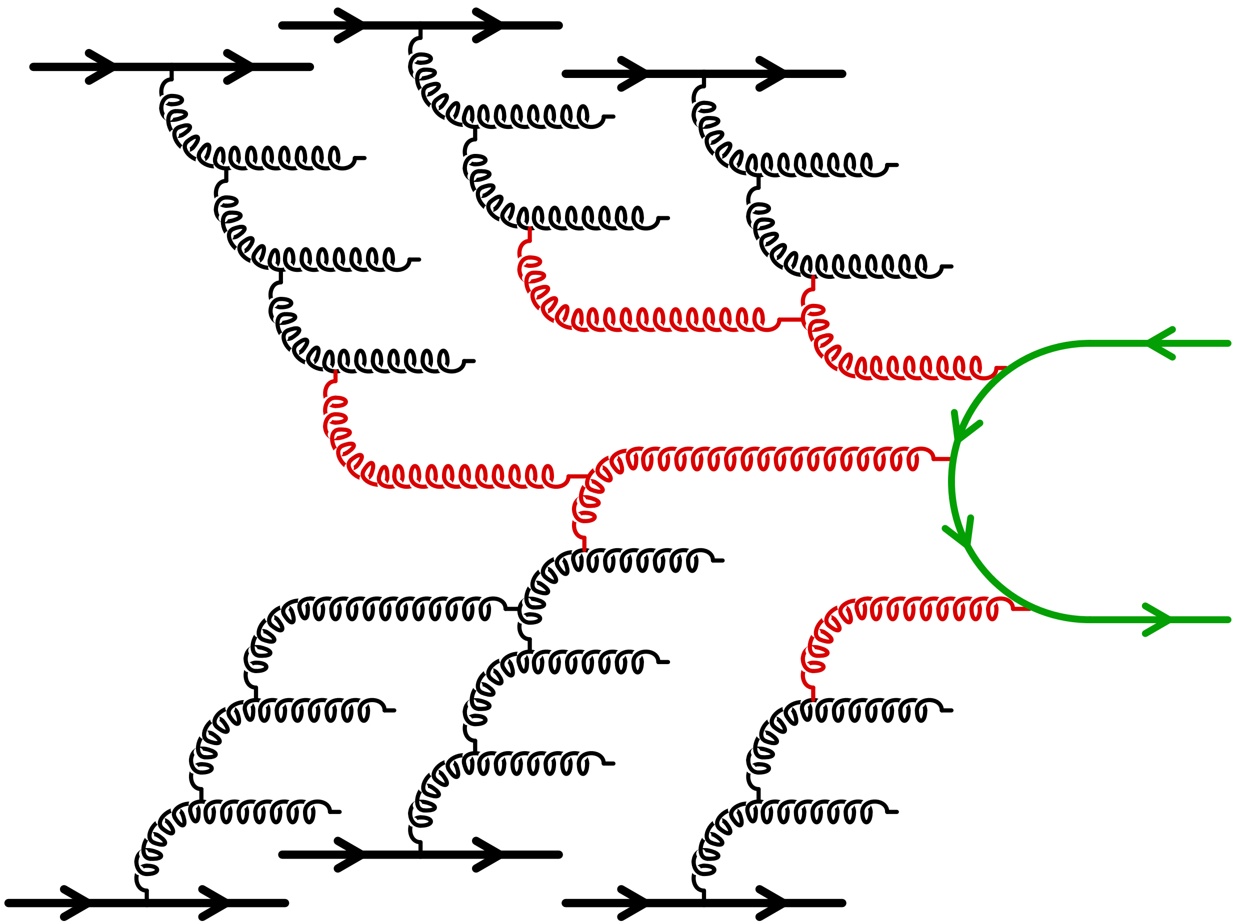
▷ find $W_{x_2}[\rho_2]$ for the second projectile

$Q\bar{Q}$ production: overview (3/4)



▷ solve the Yang-Mills equation for sources ρ_1 and ρ_2

$Q\bar{Q}$ production: overview (4/4)



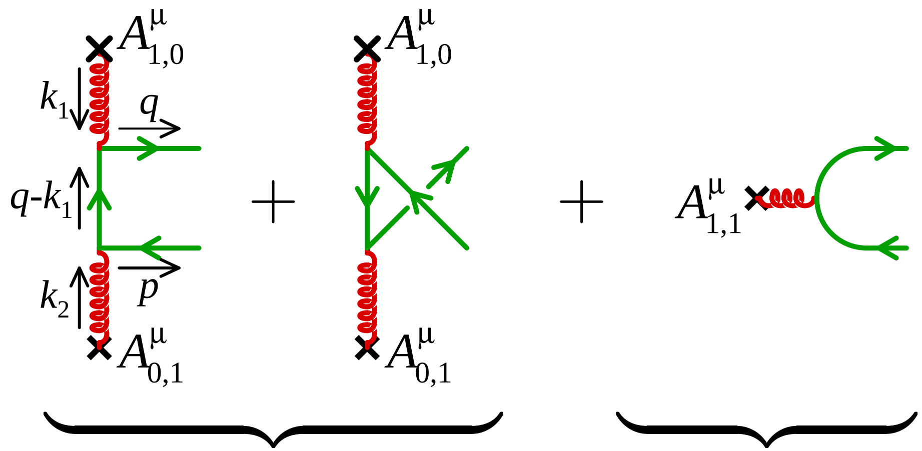
▷ find the quark propagator in the classical color field

Dilute regime: *pp* (1/5)

FG, Venugopalan (2003)

- At leading order in the hard sources ρ_1, ρ_2:

$$\mathcal{M} = \overline{u}(\vec{\boldsymbol{q}})\mathcal{T}_R v(\vec{\boldsymbol{p}}) =$$



$$= \mathcal{M}_{\rm abelian}(\vec{\boldsymbol{q}},\vec{\boldsymbol{p}}) + \mathcal{M}_{\rm 3g}(\vec{\boldsymbol{q}},\vec{\boldsymbol{p}})$$

- **Notes:**
  - $A^\mu_{1,0}$, $A^\mu_{0,1}$, $A^\mu_{1,1}$ are the color fields at order ρ_1, ρ_2, $\rho_1\rho_2$
  - $P_1 = \overline{n}$ at this order (retarded = time ordered)

Dilute regime: pp (2/5)

- Classical field in the covariant gauge at $\mathcal{O}(\rho_1\rho_2)$:

$$A^+_{1,0}(k) = 2\pi g\delta(k^-)\frac{1}{\boldsymbol{k}_\perp^2}\rho_1(\vec{\boldsymbol{k}}_\perp)\,,\; A^-_{1,0}(k) = A^i_{1,0}(k) = 0$$

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$$A^\mu_{1,1}(k) = -\frac{g}{k^2}\int\frac{d^4k_1}{(2\pi)^4}L^\mu(k,k_1)\left[A^+_{1,0}(k_1),A^-_{0,1}(k_2)\right]$$

$$L^+\equiv k^+-\boldsymbol{k}_{1\perp}^2/k^-\,,\ L^-\equiv\boldsymbol{k}_{2\perp}^2/k^+-k^-\,,\ L^i\equiv\boldsymbol{k}_2^i-\boldsymbol{k}_1^i$$

Kovchegov, Rischke (1997)

Dilute regime: pp (3/5)

- Pair production amplitude:

$$\mathcal{M}_{\rm abelian} = ig^2 \int \frac{d^4k_1}{(2\pi)^4} A^{+a}_{1,0}(k_1) A^{-b}_{0,1}(k_2)$$
$$\times \overline{u}(\vec{\boldsymbol{q}}) \left\{ \gamma^- \frac{m - \vec{\boldsymbol{\gamma}}_\perp \cdot (\vec{\boldsymbol{q}}_\perp - \vec{\boldsymbol{k}}_{1\perp})}{2q^-p^+ + (\vec{\boldsymbol{q}}_\perp - \vec{\boldsymbol{k}}_{1\perp})^2 + m^2} \gamma^+ t_a t_b \right.$$
$$\left. + \gamma^+ \frac{m + \vec{\boldsymbol{\gamma}}_\perp \cdot (\vec{\boldsymbol{p}}_\perp - \vec{\boldsymbol{k}}_{1\perp})}{2q^+p^- + (\vec{\boldsymbol{p}}_\perp - \vec{\boldsymbol{k}}_{1\perp})^2 + m^2} \gamma^- t_b t_a \right\} v(\vec{\boldsymbol{p}})$$

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$$\mathcal{M}_{3g} = ig^2 \frac{[t_a, t_b]}{(p+q)^2} \int \frac{d^4k_1}{(2\pi)^4} A^{+a}_{1,0}(k_1) A^{-b}_{0,1}(k_2)\, \overline{u}(\vec{\boldsymbol{q}})\, \not{L} v(\vec{\boldsymbol{p}})$$

Dilute regime: pp (4/5)

- In terms of ρ_1 and ρ_2, we can write:

$$\mathcal{M} \equiv g^2 \int \frac{d^2\vec{\boldsymbol{k}}_{1\perp}}{(2\pi)^2} \frac{\rho_1(\vec{\boldsymbol{k}}_{1\perp})}{k_{1\perp}} \frac{\rho_2(\vec{\boldsymbol{k}}_{2\perp})}{k_{2\perp}} \frac{m(k_1, k_2; \vec{\boldsymbol{p}}, \vec{\boldsymbol{q}})}{k_{1\perp} k_{2\perp}}$$

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- Zero transfer limit:

$$\lim_{\boldsymbol{k}_{1\perp}\to 0} m(k_1, k_2; \vec{\boldsymbol{p}}, \vec{\boldsymbol{q}}) = \lim_{\boldsymbol{k}_{2\perp}\to 0} m(k_1, k_2; \vec{\boldsymbol{p}}, \vec{\boldsymbol{q}}) = 0$$

$\triangleright$ This is crucial for collinear factorization

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- Boost invariance:

$m(k_1, k_2; \vec{\boldsymbol{p}}, \vec{\boldsymbol{q}})$ depends only on the rapidity difference $y_p - y_q$ between the quark and the antiquark

Dilute regime: pp (5/5)

- $k_\perp$-factorization of the cross-section:

$$\frac{d\sigma}{dy_p dy_q d^2\vec{\boldsymbol{p}}_\perp d^2\vec{\boldsymbol{q}}_\perp} \propto \int \frac{d^2\vec{\boldsymbol{k}}_{1\perp}}{(2\pi)^2}\frac{d^2\vec{\boldsymbol{k}}_{2\perp}}{(2\pi)^2}\delta(\vec{\boldsymbol{k}}_{1\perp}+\vec{\boldsymbol{k}}_{2\perp}-\vec{\boldsymbol{p}}_\perp-\vec{\boldsymbol{q}}_\perp)$$

$$\times\varphi_1(k_{1\perp})\varphi_2(k_{2\perp})\frac{\mathrm{Tr}\,\left|m\right|^2}{k_{1\perp}^2 k_{2\perp}^2}$$

$$\varphi(k_\perp)\equiv g^2\int d^2\vec{\boldsymbol{x}}_\perp d^2\vec{\boldsymbol{r}}_\perp e^{-i\vec{\boldsymbol{k}}_\perp\cdot\vec{\boldsymbol{r}}_\perp}\frac{\left\langle\rho(\vec{\boldsymbol{x}}_\perp+\frac{\vec{\boldsymbol{r}}_\perp}{2})\rho(\vec{\boldsymbol{x}}_\perp-\frac{\vec{\boldsymbol{r}}_\perp}{2})\right\rangle}{k_\perp^2}$$

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- At leading order in $\rho_{1,2}$, all the saturation physics goes into the distributions $\varphi_{1,2}(k_\perp)$
- $\mathrm{Tr}\,\left|m\right|^2$ is identical to the matrix element obtained by Collins & Ellis (1991) in pQCD

Semi-dense regime: pA (1/3)

Blaizot, FG, Venugopalan (work in progress)

- ρ_1 is a weak source, ρ_2 is a strong source
  $\Rightarrow$ we want the pair production amplitude to first order in ρ_1 and to all orders in ρ_2

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- $k_\perp$-factorization ?
  - it is known to work at this order for gluon production
  - does it work for $Q\bar{Q}$ production ?
  - with an universal “unintegrated gluon distribution” ?

Semi-dense regime: pA (2/3)

- Gauge field in the covariant gauge ($\partial_\mu A^\mu = 0$):

$$k^2 A^\mu_{1,\infty}(k) = \int \frac{d^4k_1}{(2\pi)^4} \Big[C_1^\mu U(k_2) + C_2^\mu V(k_2) + C_3^\mu \mathbb{1}(k_2) \Big] A^+_{1,0}(k_1)$$

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$$U(k_2) \equiv 2\pi\delta(k_2^+)\int d^2\vec{\boldsymbol{x}}_\perp e^{-i\vec{\boldsymbol{k}}_{2\perp}\cdot\vec{\boldsymbol{x}}_\perp}\mathcal{P}e^{-ig\int_{z^+} A^-_{0,1}(z^+,\vec{\boldsymbol{x}}_\perp)\cdot T}$$

$$V(k_2) \equiv 2\pi\delta(k_2^+)\int d^2\vec{\boldsymbol{x}}_\perp e^{-i\vec{\boldsymbol{k}}_{2\perp}\cdot\vec{\boldsymbol{x}}_\perp}\mathcal{P}e^{-i\frac{g}{2}\int_{z^+} A^-_{0,1}(z^+,\vec{\boldsymbol{x}}_\perp)\cdot T}$$

$$\mathbb{1}(k_2) \equiv (2\pi)^3\delta(k_2^+)\delta(\vec{\boldsymbol{k}}_{2\perp})$$

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$$\mathbb{1}(k_2) \equiv (2\pi)^3 \delta(k_2^+)\delta(\vec{\boldsymbol{k}}_{2\perp})$$

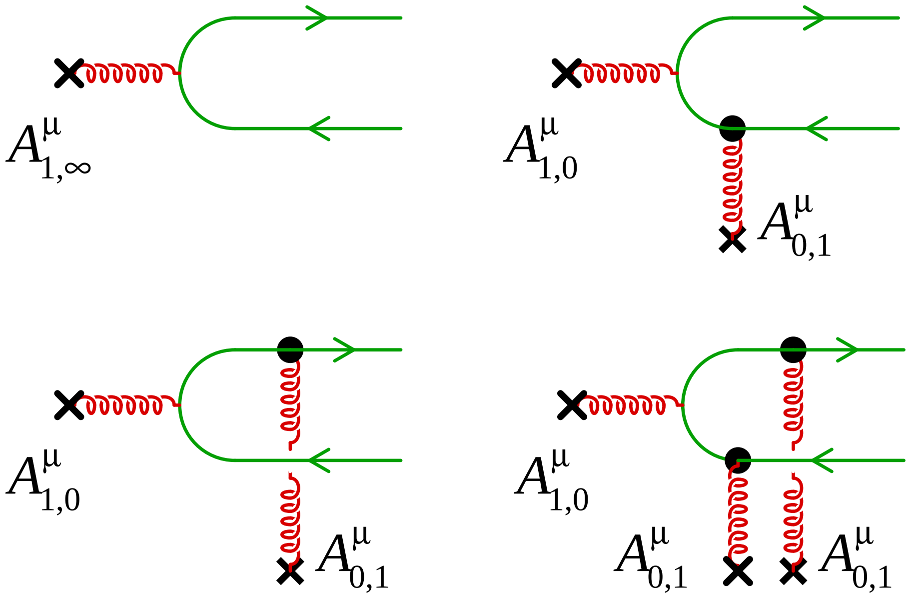
$$C_1^+ \equiv k_1^2/k^- \,,\; C_1^- \equiv -(k_2^2 + k_\perp^2)/k^+ \,,\; C_1^i \equiv -2k_1^i$$

$$C_2^+ \equiv 2k^+ \,,\; C_2^- \equiv -2k^- + 2k_\perp^2/k^+ \,,\; C_2^i \equiv 2k^i$$

$$C_3^+ \equiv -2k^+ + k_\perp^2/k^- \,,\; C_3^- \equiv 2k^- - k_\perp^2/k^+ \,,\; C_3^i \equiv 0$$

Semi-dense regime: pA (3/3)

- Diagrams for $Q\bar{Q}$ production:



Dense regime: AA (1/7)

FG, Kajantie, Lappi (work in progress)

- Alternate representation of the amplitude:

$$\overline{u}(\vec{\boldsymbol{q}})\mathcal{T}_{R}v(\vec{\boldsymbol{p}}) = \lim_{t\to+\infty}\int d^3\vec{\boldsymbol{x}}\ \phi^\dagger_{\boldsymbol{q}}(t,\vec{\boldsymbol{x}})\psi_{\boldsymbol{p}}(t,\vec{\boldsymbol{x}})$$

$$(i\partial\!\!\!/_x - gA\!\!\!/(x) - m)\psi_{\boldsymbol{p}}(x) = 0\ ,\ \psi_{\boldsymbol{p}}(t,\vec{\boldsymbol{x}}) \underset{t\to-\infty}{\longrightarrow} v(\vec{\boldsymbol{p}})e^{ip\cdot x}$$

$$\phi_{\boldsymbol{q}}(t,\vec{\boldsymbol{x}}) = u(\vec{\boldsymbol{q}})e^{-iq\cdot x}$$

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- On a surface of constant proper time:

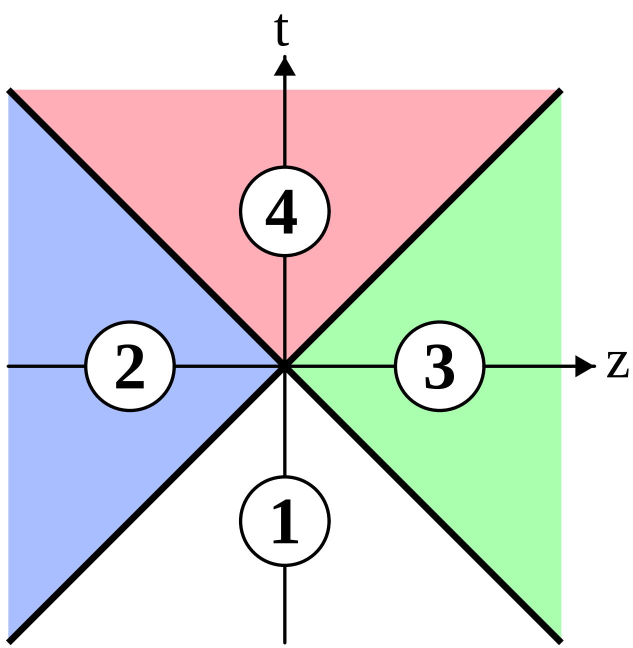
$$\overline{u}(\vec{\boldsymbol{q}})\mathcal{T}_{_R}v(\vec{\boldsymbol{p}}) = \lim_{\tau\to+\infty}\tau\int d\eta d^2\vec{\boldsymbol{x}}_\perp\;\widetilde{\phi}^\dagger_{\boldsymbol{q}}(\tau,\eta,\vec{\boldsymbol{x}}_\perp)\widetilde{\psi}_{\boldsymbol{p}}(\tau,\eta,\vec{\boldsymbol{x}}_\perp)$$

$$\widetilde{\phi}_{\boldsymbol{q}} \equiv e^{-\frac{\eta}{2}\gamma^0\gamma^3}\phi_{\boldsymbol{q}}\;,\;\; \widetilde{\psi}_{\boldsymbol{p}} \equiv e^{-\frac{\eta}{2}\gamma^0\gamma^3}\psi_{\boldsymbol{p}}$$

$$t = \tau\cosh(\eta)\;,\;\; z = \tau\sinh(\eta)$$

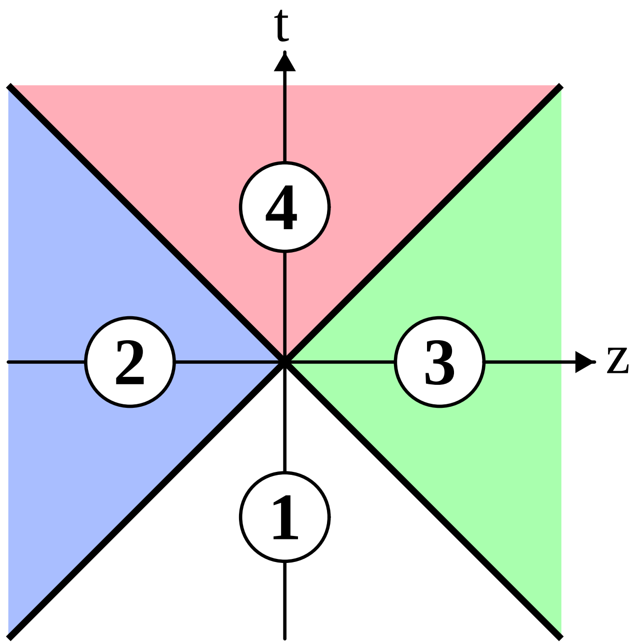
Dense regime: AA (2/7)

- Space-time structure of the classical color field:



Dense regime: AA (2/7)

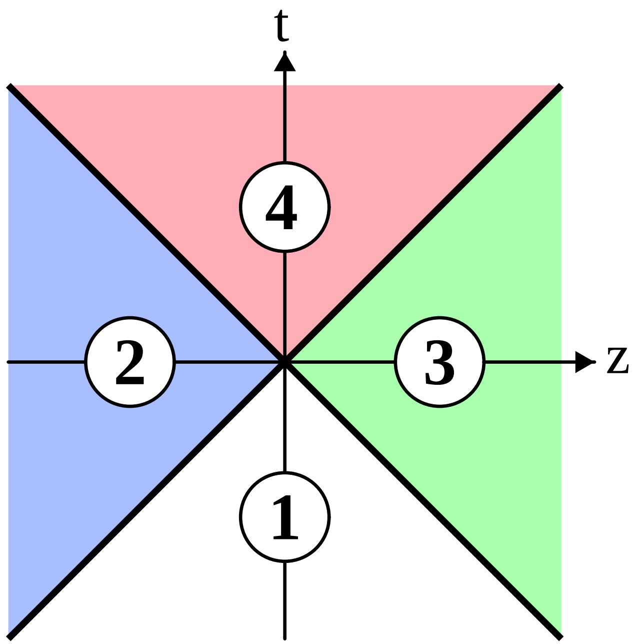
- Space-time structure of the classical color field:



- Region 1: $A^\mu = 0$
- Region 2: $A^\pm = 0$,
  $A^i = \frac{i}{g} U_1 \nabla_\perp^i U_1^\dagger$
- Region 3: $A^\pm = 0$,
  $A^i = \frac{i}{g} U_2 \nabla_\perp^i U_2^\dagger$
- Region 4: $A^\mu \neq 0$

Dense regime: AA (2/7)

- Space-time structure of the classical color field:



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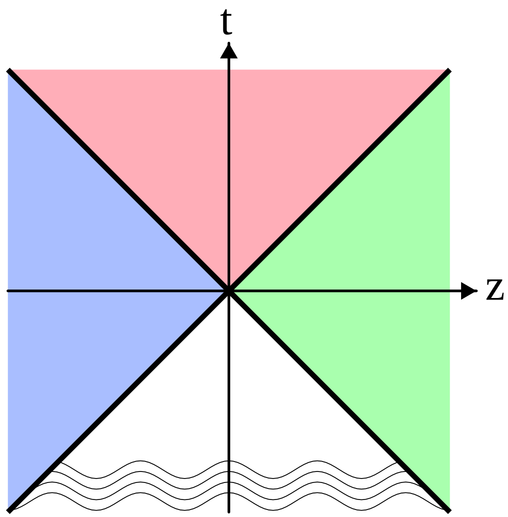
  $A^i = \frac{i}{g} U_2 \nabla_\perp^i U_2^\dagger$
- Region 4: $A^\mu \neq 0$

- **Notes:**
  - $U_{1,2}(\vec{\boldsymbol{x}}_\perp) = \exp(-ig\frac{1}{\nabla_\perp^2}\rho_{1,2})$
  - In the region 4, A^μ is known only numerically
    Krasnitz, Venugopalan (2000,2001), Lappi (2003)

Dense regime: AA (3/7)

- Propagation through region 1:

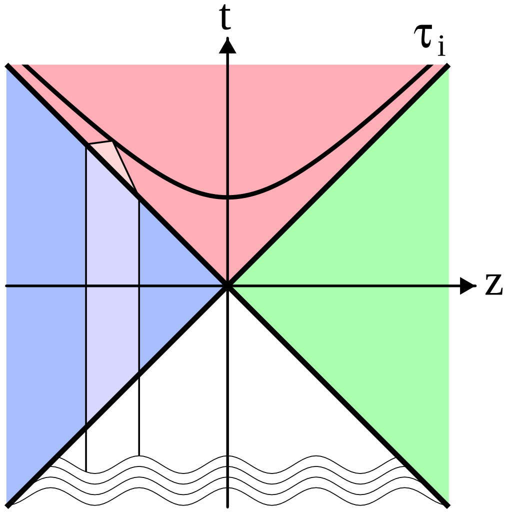


▷ trivial because there is no background field

$$\psi_{\boldsymbol{p}}(x) = v(\boldsymbol{p})e^{ip\cdot x}$$

Dense regime: AA (4/7)

- Propagation through region 2:



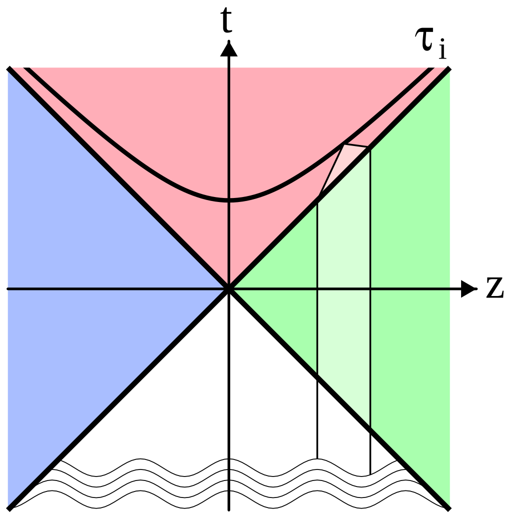
$\triangleright$ Pure gauge background field:

$$\widetilde{\psi}^{+}_{\boldsymbol{p}}(\tau_i,\eta,\vec{\boldsymbol{x}}_\perp)=\frac{1}{\omega_{\boldsymbol{p}}}\int_{\vec{\boldsymbol{k}}_\perp} e^{i\vec{\boldsymbol{k}}_\perp\cdot\vec{\boldsymbol{x}}_\perp}\, e^{\frac{y_p-\eta}{2}}\, e^{i\frac{\tau_i}{2}\frac{\omega^2_{\boldsymbol{k}}}{\omega_{\boldsymbol{p}}}e^{y_p-\eta}}\mathcal{F}_+(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{k}}_\perp;\vec{\boldsymbol{p}}_\perp)$$

with $\quad \mathcal{F}_+\equiv\frac{\gamma^-}{\sqrt{2}}U_1(\vec{\boldsymbol{x}}_\perp)(m+\vec{\boldsymbol{k}}_\perp\cdot\vec{\gamma}_\perp)\widetilde{U_1}^\dagger(\vec{\boldsymbol{p}}_\perp+\vec{\boldsymbol{k}}_\perp)v(\vec{\boldsymbol{p}}_\perp)$

Dense regime: AA (5/7)

- Propagation through region 3:



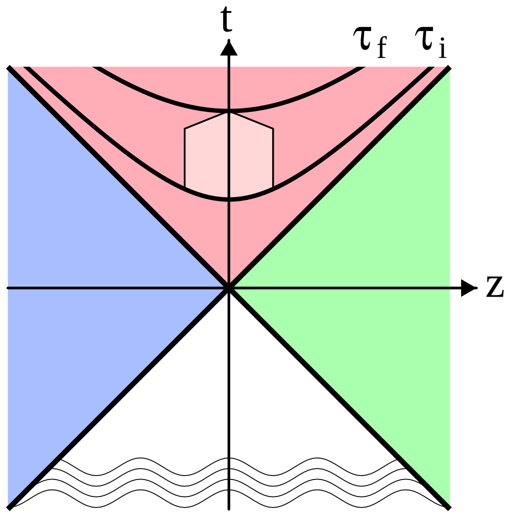
▷ Pure gauge background field:

$$\widetilde{\psi}^{-}_{\boldsymbol{p}}(\tau_i, \eta, \vec{\boldsymbol{x}}_\perp) = \frac{1}{\omega_{\boldsymbol{p}}} \int_{\vec{\boldsymbol{k}}_\perp} e^{i\vec{\boldsymbol{k}}_\perp\cdot\vec{\boldsymbol{x}}_\perp}\, e^{\frac{\eta - y_p}{2}}\, e^{i\frac{\tau_i}{2}\frac{\omega^2_{\boldsymbol{k}}}{\omega_{\boldsymbol{p}}}e^{\eta - y_p}} \mathcal{F}_-(\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{k}}_\perp; \vec{\boldsymbol{p}}_\perp)$$

with $\quad \mathcal{F}_- \equiv \frac{\gamma^+}{\sqrt{2}} U_2(\vec{\boldsymbol{x}}_\perp)(m + \vec{\boldsymbol{k}}_\perp \cdot \vec{\gamma}_\perp)\widetilde{U_2}^\dagger(\vec{\boldsymbol{p}}_\perp + \vec{\boldsymbol{k}}_\perp)v(\vec{\boldsymbol{p}}_\perp)$

Dense regime: AA (6/7)

- Propagation through region 4:



$$\partial_\tau \widetilde{\psi}_{\boldsymbol{p}}(\tau,\eta,\vec{\boldsymbol{x}}_\perp) = \Big[-\frac{1}{2\tau} - \frac{\gamma^0\gamma^3}{\tau}(\partial_\eta + igA_\eta) + \gamma^0 \vec{\boldsymbol{\gamma}}_\perp \cdot (\vec{\boldsymbol{\nabla}}_\perp + ig\vec{\boldsymbol{A}}_\perp) - i\gamma^0 m \Big] \widetilde{\psi}_{\boldsymbol{p}}(\tau,\eta,\vec{\boldsymbol{x}}_\perp)$$

▷ initial condition: $\widetilde{\psi}_{\boldsymbol{p}}(\tau_i) = \widetilde{\psi}^+_{\boldsymbol{p}}(\tau_i) + \widetilde{\psi}^-_{\boldsymbol{p}}(\tau_i)$

Dense regime: AA (7/7)

- Boost invariance:

  ▷ $\widetilde{\psi}_{\boldsymbol{p}}(\tau, \eta, \vec{\boldsymbol{x}}_\perp)$ depends only on $\eta - y_p$ (this is manifest at τ_i, and A_η, $\vec{\boldsymbol{A}}_\perp$ do not depend on $\eta \Rightarrow$ true at any τ)

  ▷ $\widetilde{\phi}_{\boldsymbol{q}}(\tau, \eta, \vec{\boldsymbol{x}}_\perp)$ depends only on $\eta - y_q$

  ▷ after integrating out η: depends only on $y_p - y_q$

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- Main difficulty: the modes in $\exp(\pm(p+1/2)\eta)$ have an instability in $\tau^p \Rightarrow$ one cannot replace $\exp(i\frac{\tau_i}{2}\frac{\omega_{\boldsymbol{k}}^2}{\omega_{\boldsymbol{p}}}e^{\pm\eta})$ by 1 in the initial condition, even if $\tau_i \to 0$

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  $\triangleright$ Physical reason: if one wants to reach the time τ_f, causality implies that at τ_i we describe correctly a range $|\eta| \le \eta_i$ with $\eta_i = \ln(\tau_f/\tau_i) \Rightarrow$ the size of $\tau_i \exp(\eta_i)$ is fixed when $\tau_i \to 0$

To be continued...

Conclusions

- At leading order in the hard sources
  - The classical field approach is equivalent to $k_\perp$-factorized pQCD
  - Saturation arise only via the gluon distribution
- The method can be extended in order to include higher order corrections in the hard sources
  - analytically for pA
  - numerically for AA
- All-orders calculation
  - Can be reduced to solving a Dirac equation with specific initial conditions in a known external field